

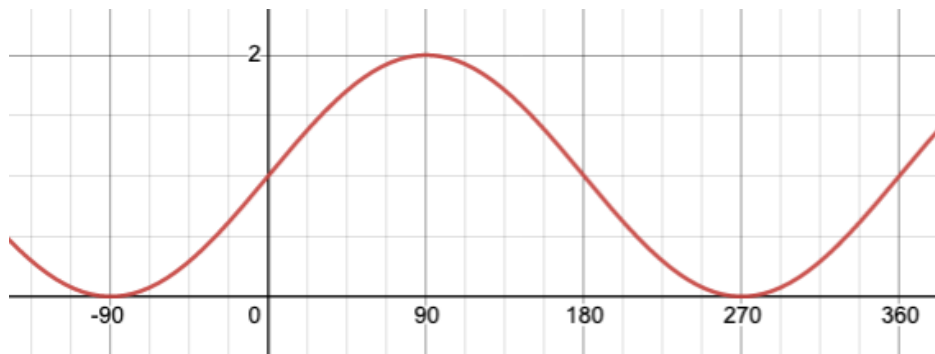
# 4.2 - Translations of Sine and Cosine

1/23

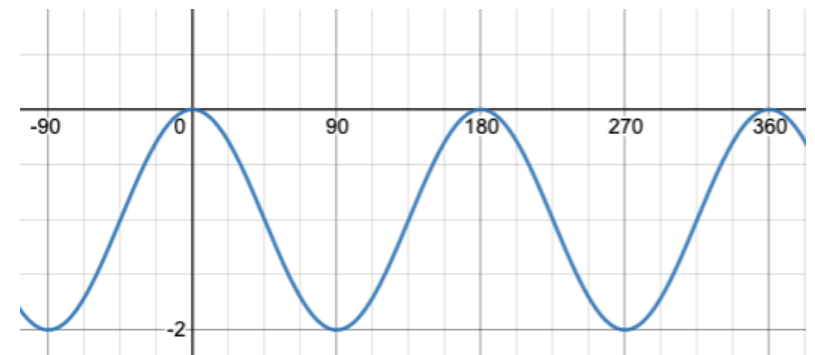
## Warmup

Graph the equations.

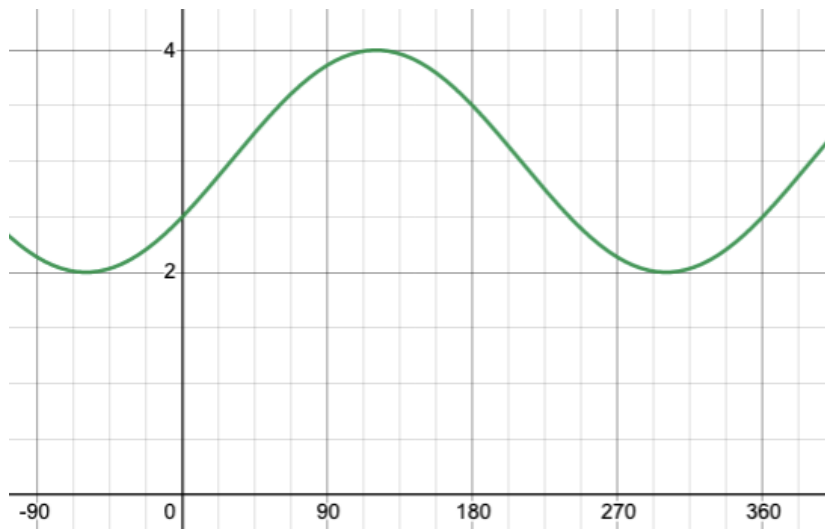
1)  $y = \sin(x) + 1$



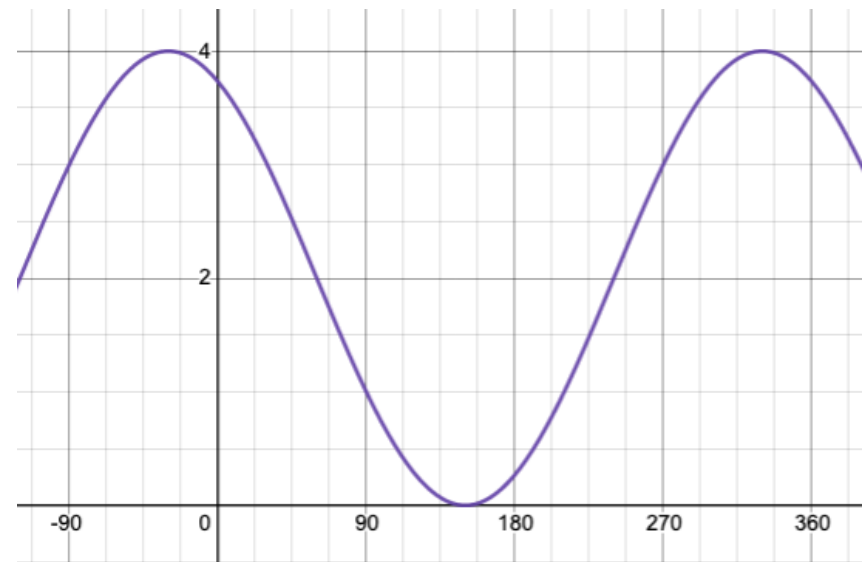
2)  $y = \cos(2x) - 1$



3)  $y = \sin(x - 30^\circ) + 3$



4)  $y = 2 \cos(x + 30^\circ) + 2$



# 4.2 - Translations of Sine and Cosine

## Standard Form Equation

2/23

$$y = A \sin \left[ B \left( x \pm \frac{C}{B} \right) \right]$$

Amplitude  $|A|$

Period  $p = \frac{2\pi}{B}$

Phase Shift  $\frac{C}{B}$

## 4.2 - Translations of Sine and Cosine

**Constructing the Function**  $y = A \cos(Bx \pm C) + k$  <sup>3/23</sup>

Amplitude  $|A| = \left| \frac{\text{max} - \text{min}}{2} \right|$

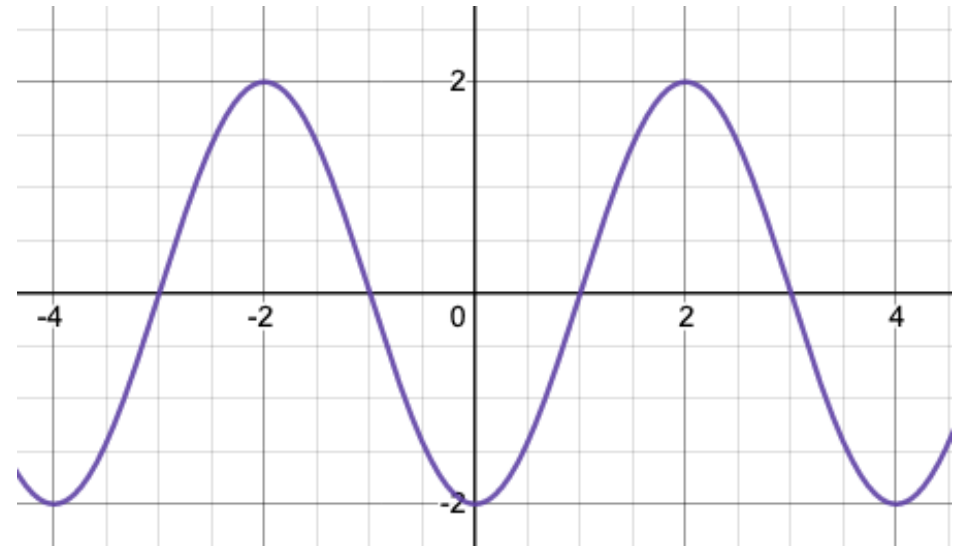
$$= \left| \frac{4}{2} \right| = 2$$

Period  $p = \frac{2\pi}{B} = 4$

$$B = \frac{\pi}{2}$$

Phase Shift (cos)  $-\frac{C}{B} = -\frac{C}{\frac{\pi}{2}} = 2 \quad C = -\pi$

Vertical Shift  $k = 0$



$$y = 2 \cos\left(\frac{\pi}{2}x - \pi\right)$$

## 4.2 - Translations of Sine and Cosine

**Constructing the Function**  $y = A \sin(Bx \pm C) + k$  4/23

Amplitude  $|A| = \left| \frac{\text{max} - \text{min}}{2} \right|$

$$= \left| \frac{6}{2} \right| = 3$$

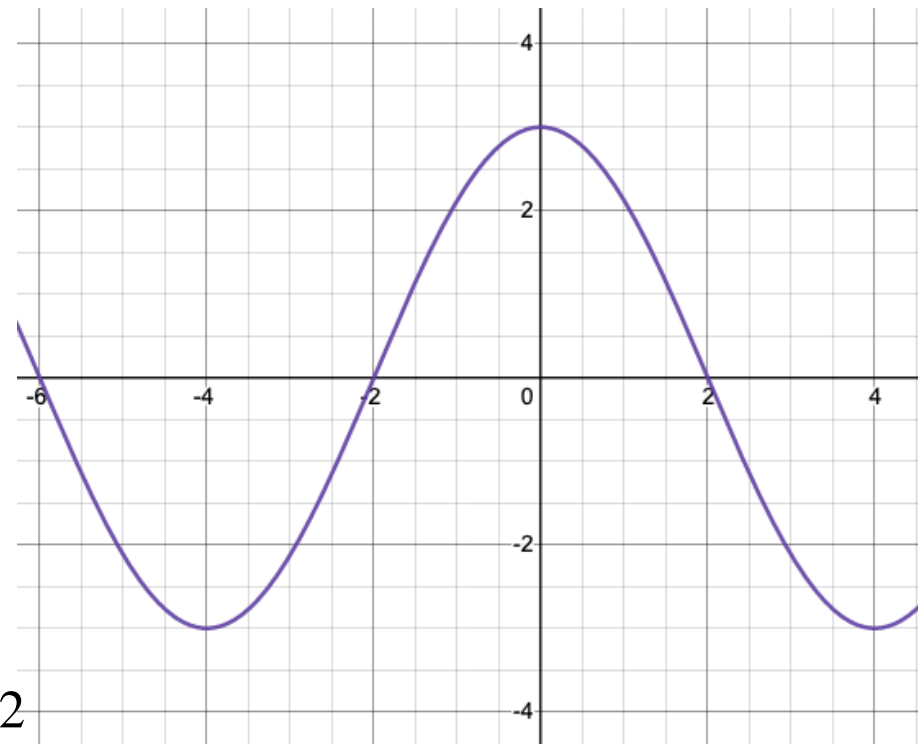
Period  $p = \frac{2\pi}{B} = 8$

$$B = \frac{\pi}{4}$$

Phase Shift (sin)  $-\frac{C}{B} = -\frac{C}{\frac{\pi}{4}} = -2$

$$C = \frac{\pi}{2}$$

Vertical Shift  $k = 0$



$$y = 3 \sin\left(\frac{\pi}{4}x + \frac{\pi}{2}\right)$$



# Graphing Tan, Cot, Sec, Csc

## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

5/23

$$\boxed{\tan x = \frac{\sin x}{\cos x}} \quad \cot x = \frac{\cos x}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \csc x = \frac{1}{\sin x}$$

x-intercepts when  $\sin x = 0$ .

$$x = n\pi$$

Vertical asymptote when  $\cos x = 0$ .

$$x = \frac{(2n + 1)\pi}{2}$$

Domain all real numbers except when  $\cos x = 0$ .

$$x \neq \frac{(2n + 1)\pi}{2}$$

Range is set of all real numbers.

$$(-\infty, \infty)$$

$y = \tan x$  has period  $\pi$ .

$y = \tan x$  is an odd function.

$$\tan(-x) = -\tan x$$

## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

6/23

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \csc x = \frac{1}{\sin x}$$

x-intercepts when  $\sin x = 0$ .

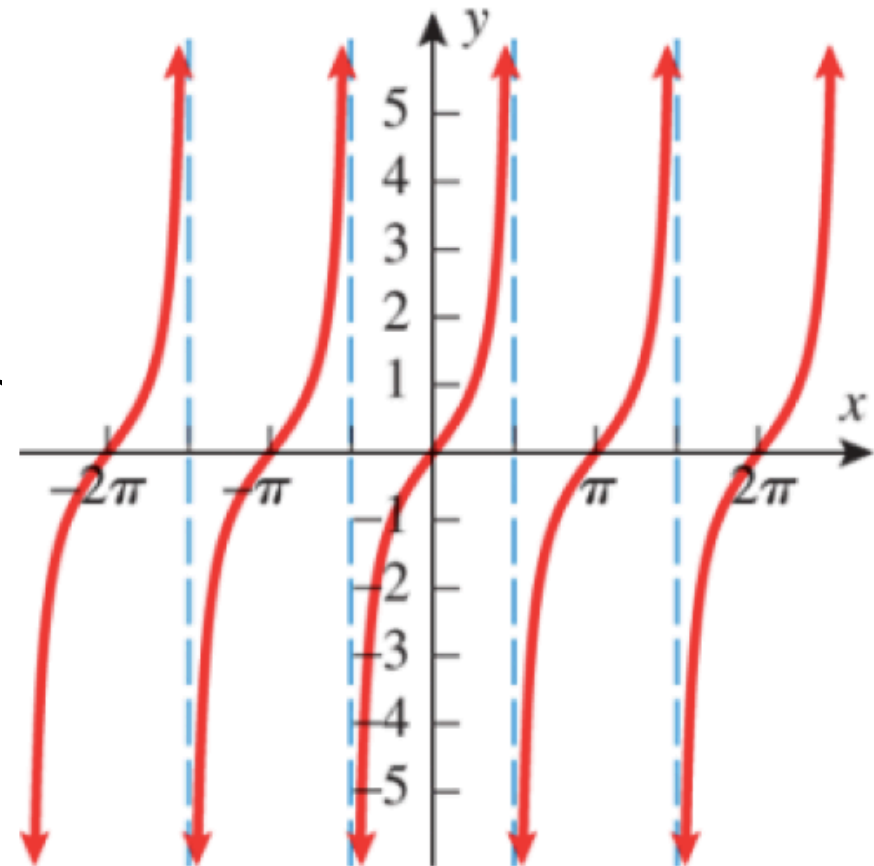
Vertical asymptote when  $\cos x = 0$ .

Domain all real numbers except when  $\cos x = 0$ .

Range is set of all real numbers.

$y = \tan x$  has period  $\pi$ .

$y = \tan x$  is an odd function.



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

7/23

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

x-intercepts when  $\cos x = 0$ .

$$x = \frac{(2n + 1)\pi}{2}$$

Vertical asymptote when  $\sin x = 0$ .

$$x = n\pi$$

Domain all real numbers except when  $\sin x = 0$ .

$$x \neq n\pi$$

Range is set of all real numbers.

$$(-\infty, \infty)$$

$y = \cot x$  has period  $\pi$ .

$y = \cot x$  is an odd function.

$$\cot(-x) = -\cot x$$



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

8/23

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

x-intercepts when  $\cos x = 0$ .

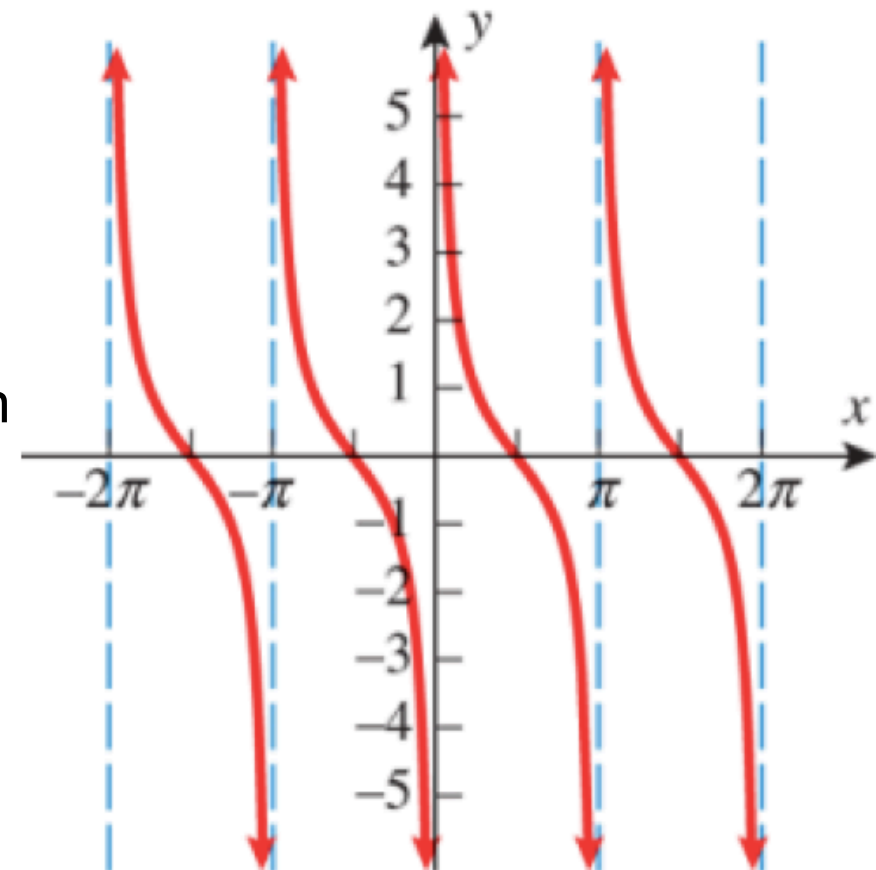
Vertical asymptote when  $\sin x = 0$ .

Domain all real numbers except when  $\sin x = 0$ .

Range is set of all real numbers.

$y = \cot x$  has period  $\pi$ .

$y = \cot x$  is an odd function.



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

9/23

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$\sec x$  has no x-intercepts.

$$\frac{1}{\cos x} \neq 0$$

Vertical asymptote when  $\cos x = 0$ .

$$x = \frac{(2n + 1)\pi}{2}$$

Domain all real numbers except when  $\cos x = 0$ .

$$x \neq \frac{(2n + 1)\pi}{2}$$

Range is  $(-\infty, -1] \cup [1, \infty)$ .

$y = \sec x$  has period  $2\pi$ .

$y = \sec x$  is an even function.

$$\sec(-x) = \sec x$$

## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

10/23

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$\sec x$  has no x-intercepts.

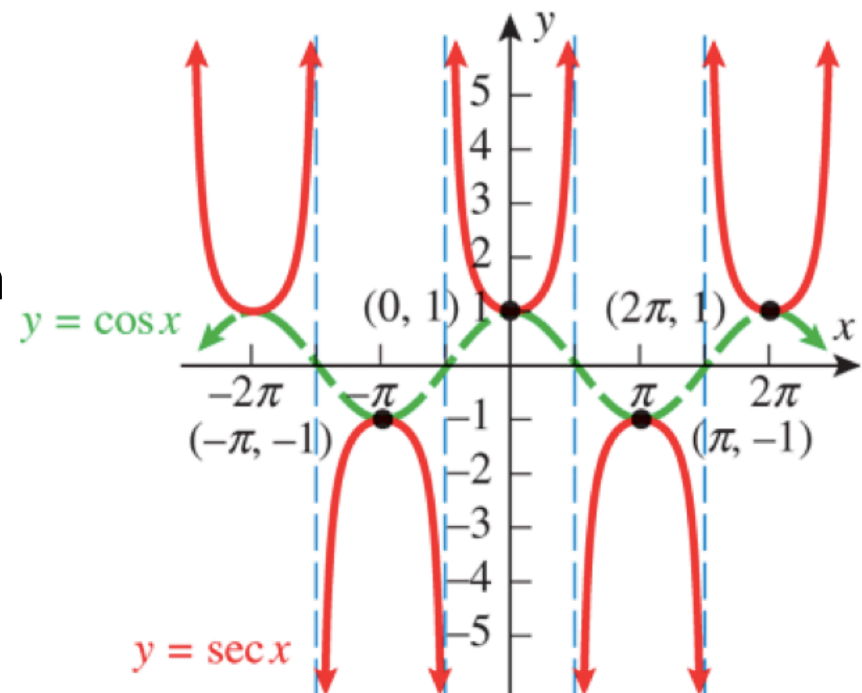
Vertical asymptote when  $\cos x = 0$ .

Domain all real numbers except when  $\cos x = 0$ .

Range is  $(-\infty, -1] \cup [1, \infty)$ .

$y = \sec x$  has period  $2\pi$ .

$y = \sec x$  is an even function.



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

11/23

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x} \quad \sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$\csc x$  has no x-intercepts.

$$\frac{1}{\sin x} \neq 0$$

Vertical asymptote when  $\sin x = 0$ .

$$x = n\pi$$

Domain all real numbers except when  $\sin x = 0$ .

$$x \neq n\pi$$

Range is  $(-\infty, -1] \cup [1, \infty)$ .

$y = \csc x$  has period  $2\pi$ .

$y = \csc x$  is an odd function.

$$\csc(-x) = -\csc x$$

## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

12/23

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x} \quad \sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$\csc x$  has no x-intercepts.

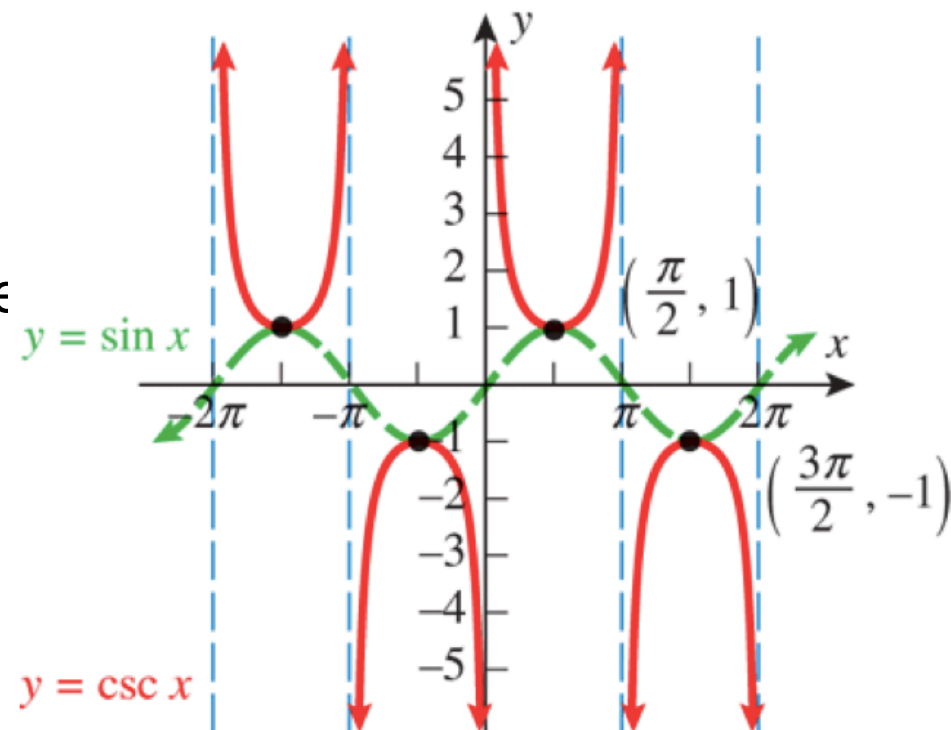
Vertical asymptote when  $\sin x = 0$ .

Domain all real numbers except where  $\sin x = 0$ .

Range is  $(-\infty, -1] \cup [1, \infty)$ .

$y = \csc x$  has period  $2\pi$ .

$y = \csc x$  is an odd function.



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

### Graphing Function $y = A \tan(Bx)$

13/23

a. Graph  $y = -3 \tan(2x)$  on the interval  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

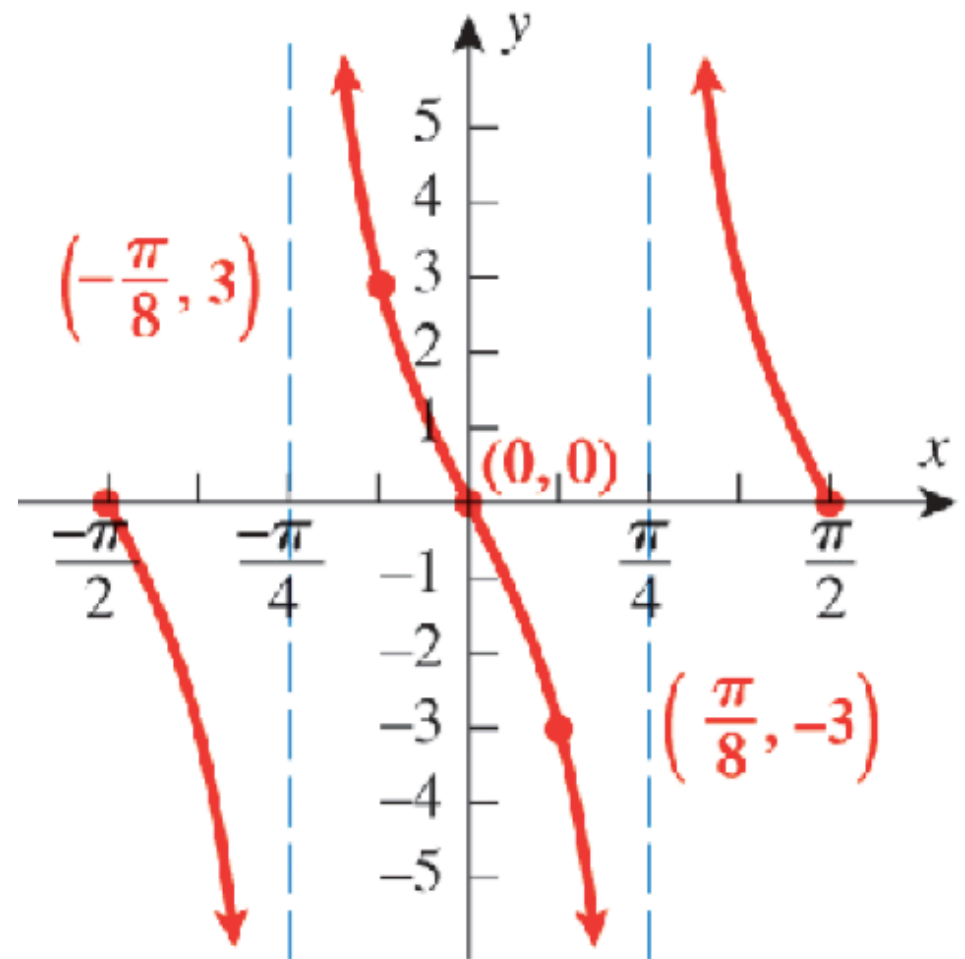
Period  $p = \frac{\pi}{B} = \frac{\pi}{2}$

Vertical Asymptote

$$Bx = -\frac{\pi}{2} \quad \text{to} \quad Bx = \frac{\pi}{2}$$
$$x = -\frac{\pi}{4} \quad \quad \quad x = \frac{\pi}{4}$$

x-intercept  $Bx = 0$

$$x = 0$$



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

**Graphing Function**  $y = A \tan(Bx)$

14/23

b. Graph  $y = \frac{1}{3} \tan\left(\frac{1}{2}x\right)$  on the interval  $-\pi \leq x \leq \pi$

Period  $p = \frac{\pi}{B} = 2\pi$

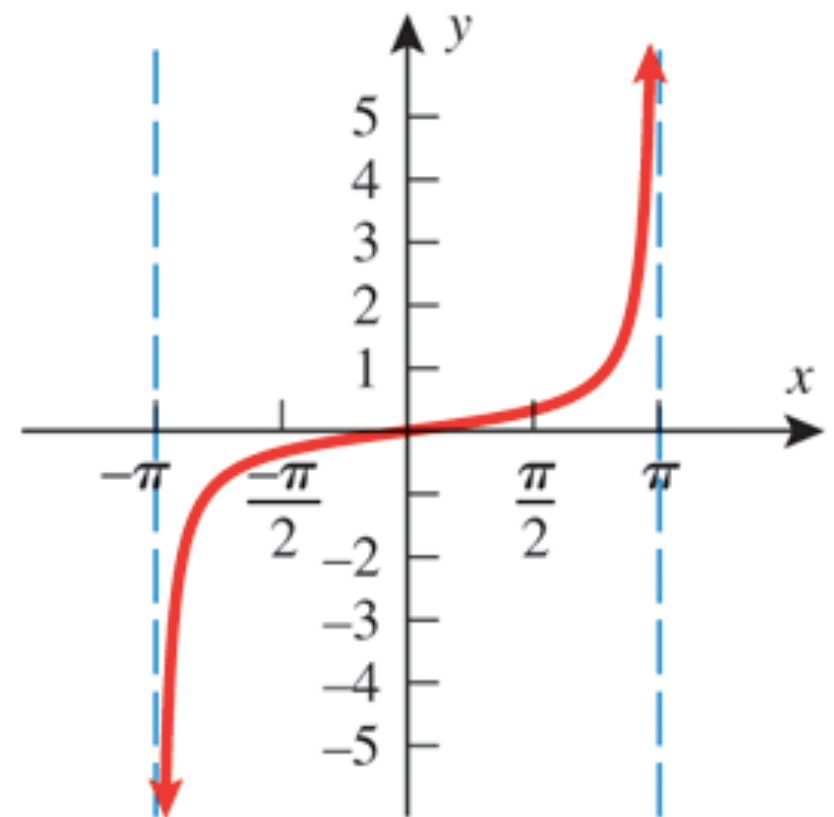
Vertical Asymptote

$$Bx = -\frac{\pi}{2} \quad \text{to} \quad Bx = \frac{\pi}{2}$$

$$x = -\pi \quad x = \pi$$

x-intercept  $Bx = 0$

$$x = 0$$



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

**Graphing Function**  $y = A \cot(Bx)$

15/23

c. Graph  $y = 4 \cot\left(\frac{1}{2}x\right)$  on the interval  $-2\pi \leq x \leq 2\pi$

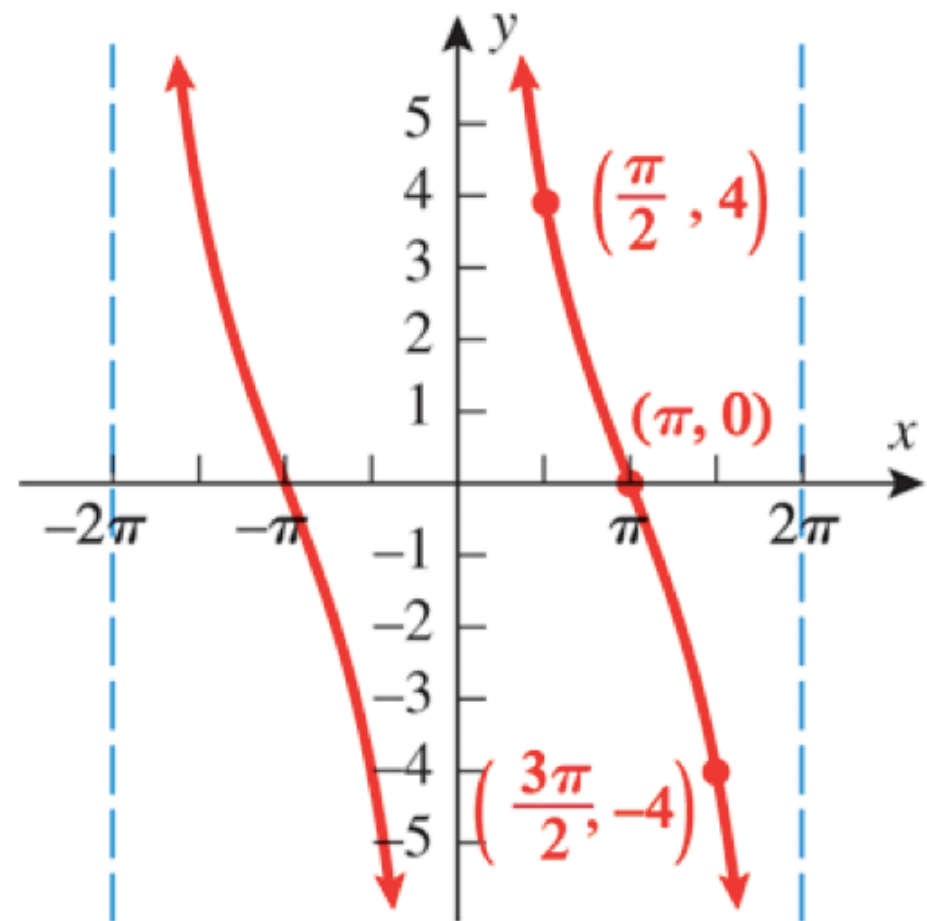
Period  $p = \frac{\pi}{B} = 2\pi$

Vertical Asymptote

$$Bx = 0 \quad \text{to} \quad Bx = \pi$$

$$x = 0 \quad \quad \quad x = 2\pi$$

x-intercept  $Bx = \frac{\pi}{2}$   
 $x = \pi$





## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

### Graphing Function $y = A \cot(Bx)$

16/23

d. Graph  $y = 2 \cot(2x)$  on the interval  $-\pi \leq x \leq \pi$

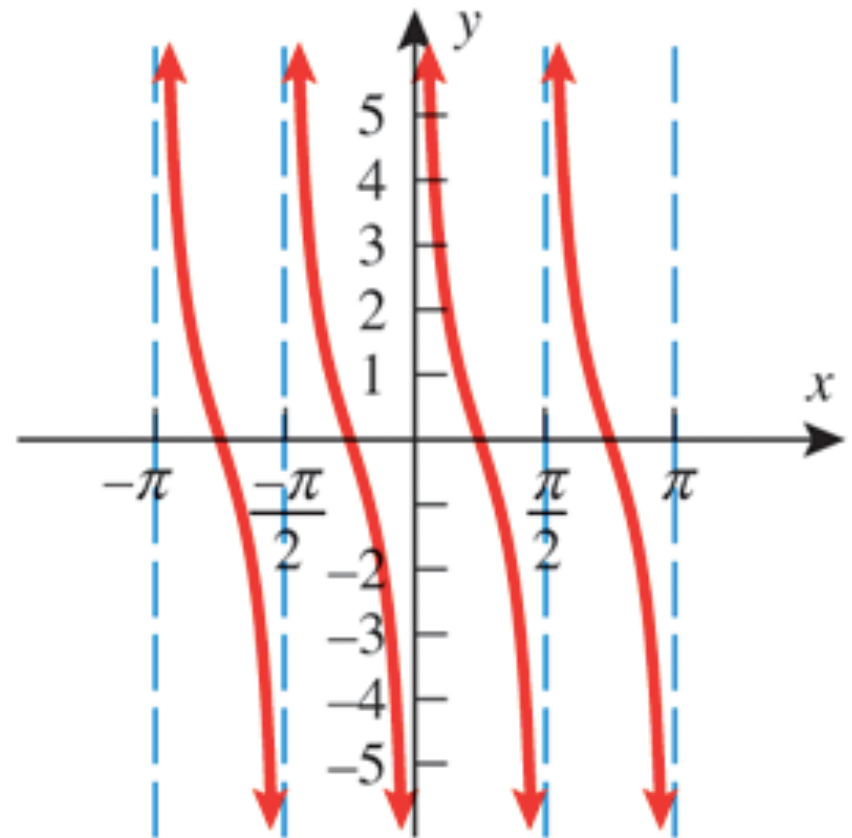
Period  $p = \frac{\pi}{B} = \frac{\pi}{2}$

Vertical Asymptote

$$Bx = 0 \quad \text{to} \quad Bx = \pi$$
$$x = 0 \quad \quad \quad x = \frac{\pi}{2}$$

x-intercept  $Bx = \frac{\pi}{2}$

$$x = \frac{\pi}{4}$$



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

**Graphing Function**  $y = A \sec(Bx)$

17/23

Graph  $y = 2 \sec(\pi x)$  on the interval  $-2 \leq x \leq 2$

Use  $y = 2 \cos(\pi x)$  as a guide.

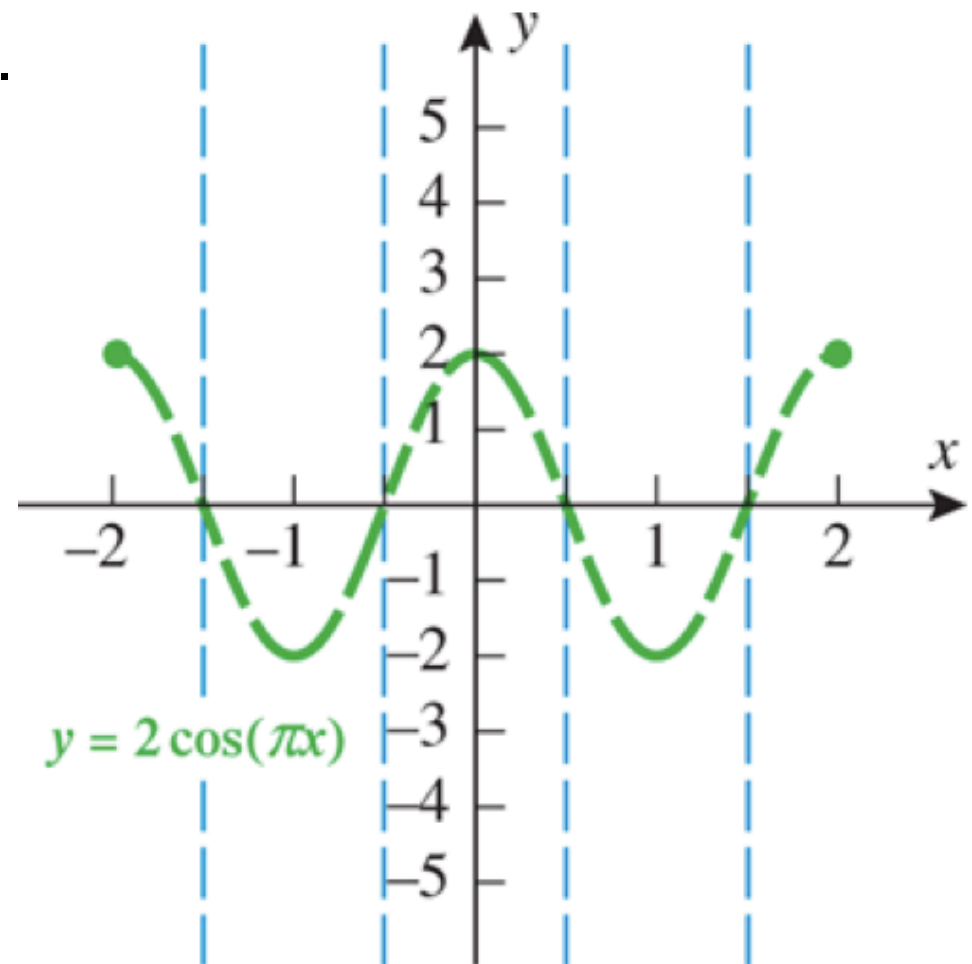
Amplitude  $|A| = 2$

Period  $p = \frac{2\pi}{\pi} = 2$

Phase Shift

$\pi x = 0$  to  $\pi x = 2\pi$

$x = 0$   $x = 2$



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

### Graphing Function $y = A \sec(Bx)$

18/23

Graph  $y = 2 \sec(\pi x)$  on the interval  $-2 \leq x \leq 2$

Use  $y = 2 \cos(\pi x)$  as a guide.

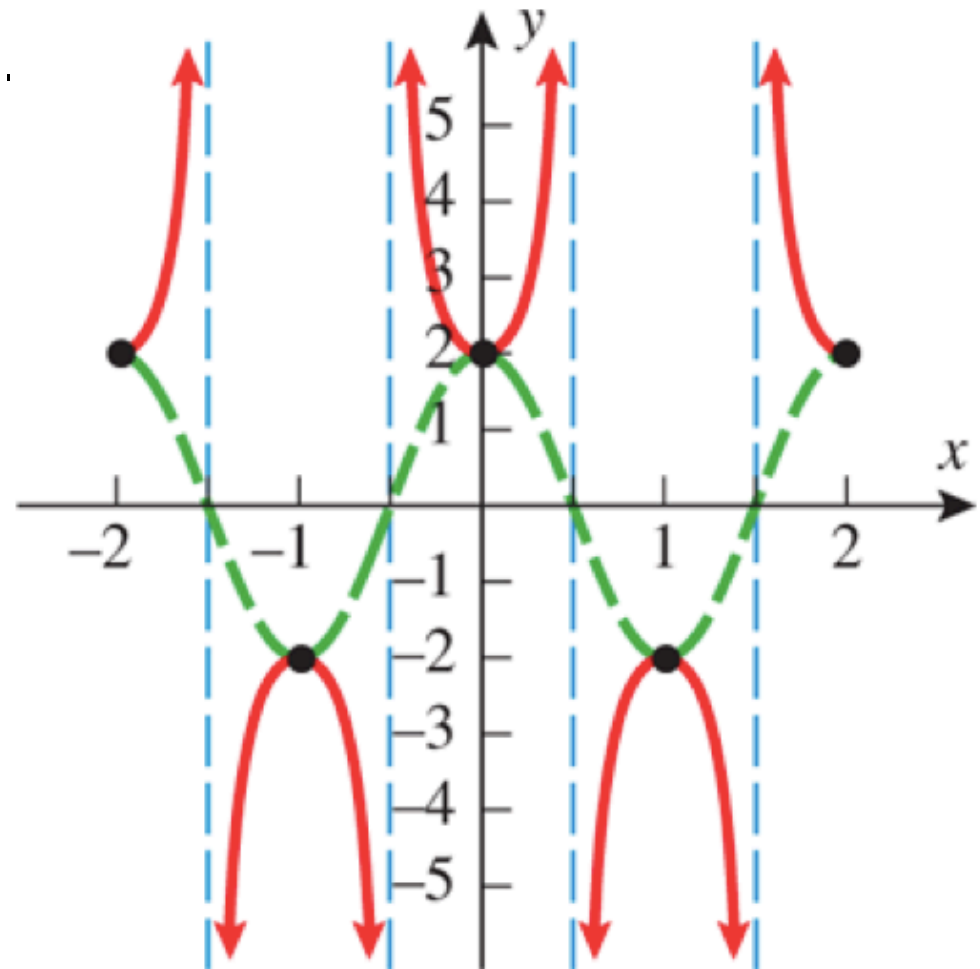
Amplitude  $|A| = 2$

Period  $p = \frac{2\pi}{\pi} = 2$

Phase Shift

$\pi x = 0$  to  $\pi x = 2\pi$

$x = 0$  to  $x = 2$



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

### Graphing Function $y = A \sec(Bx)$

19/23

a. Graph  $y = -\sec(2\pi x)$  on the interval  $-1 \leq x \leq 1$

Use  $y = -\cos(2\pi x)$  as a guide.

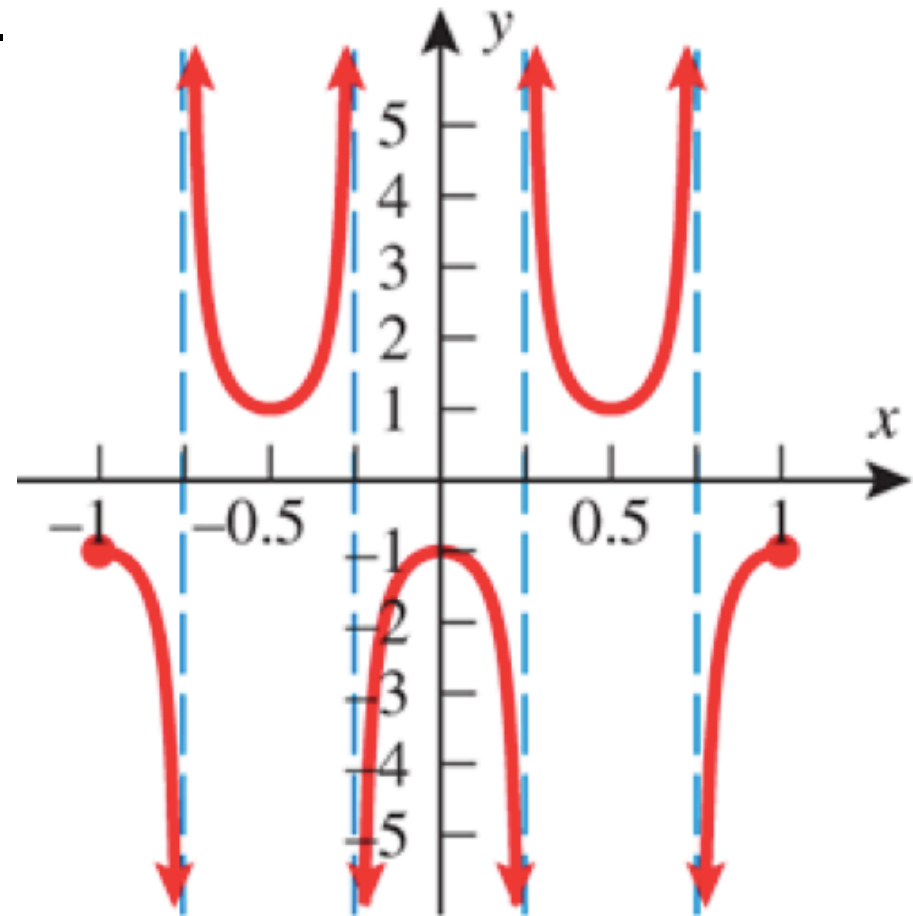
Amplitude  $|A| = 1$

Period  $p = \frac{2\pi}{2\pi} = 1$

Phase Shift

$$2\pi x = 0 \quad \text{to} \quad 2\pi x = 2\pi$$

$$x = 0 \quad \quad \quad x = 1$$



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

**Graphing Function**  $y = A \csc(Bx)$

20/23

b. Graph  $y = -3 \csc(2\pi x)$  on the interval  $-1 \leq x \leq 1$

Use  $y = -3 \sin(2\pi x)$  as a guide.

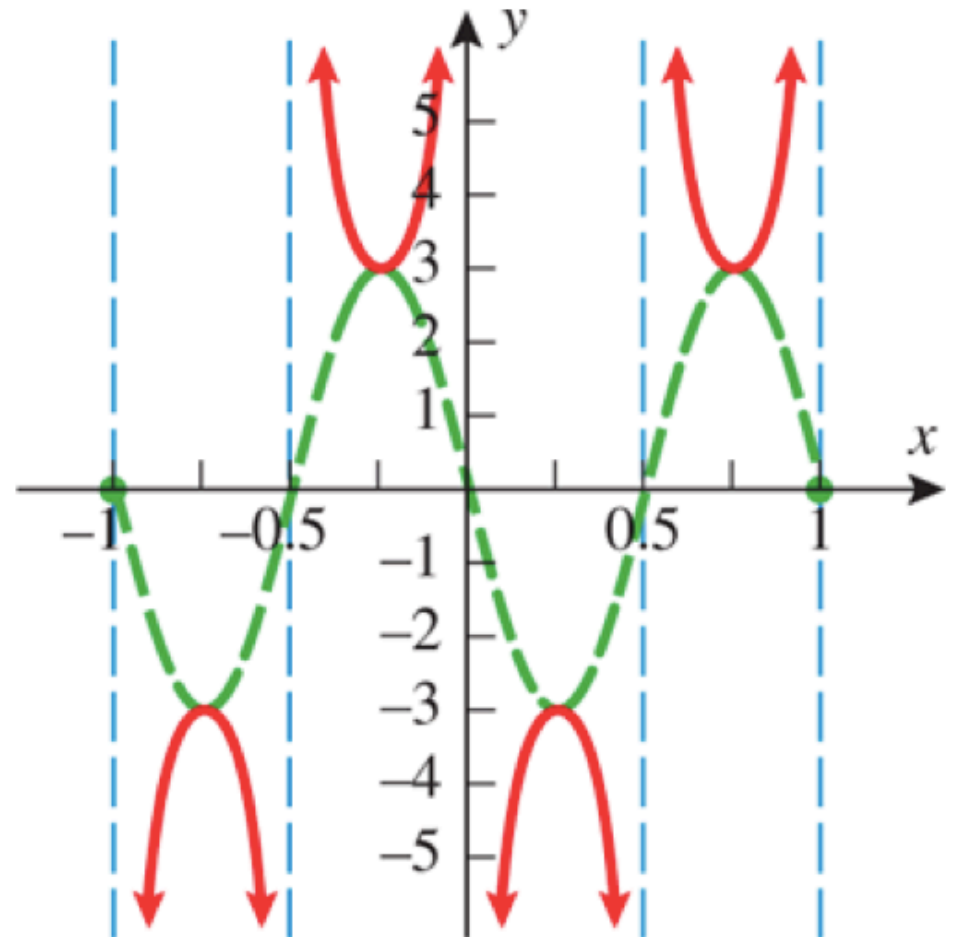
Amplitude  $|A| = 3$

Period  $p = \frac{2\pi}{2\pi} = 1$

Phase Shift

$2\pi x = 0$  to  $2\pi x = 2\pi$

$x = 0$  to  $x = 1$



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

**Graphing Function**  $y = A \csc(Bx)$

21/23

c. Graph  $y = \frac{1}{2} \csc(\pi x)$  on the interval  $-1 \leq x \leq 1$

Use  $y = \frac{1}{2} \sin(\pi x)$  as a guide.

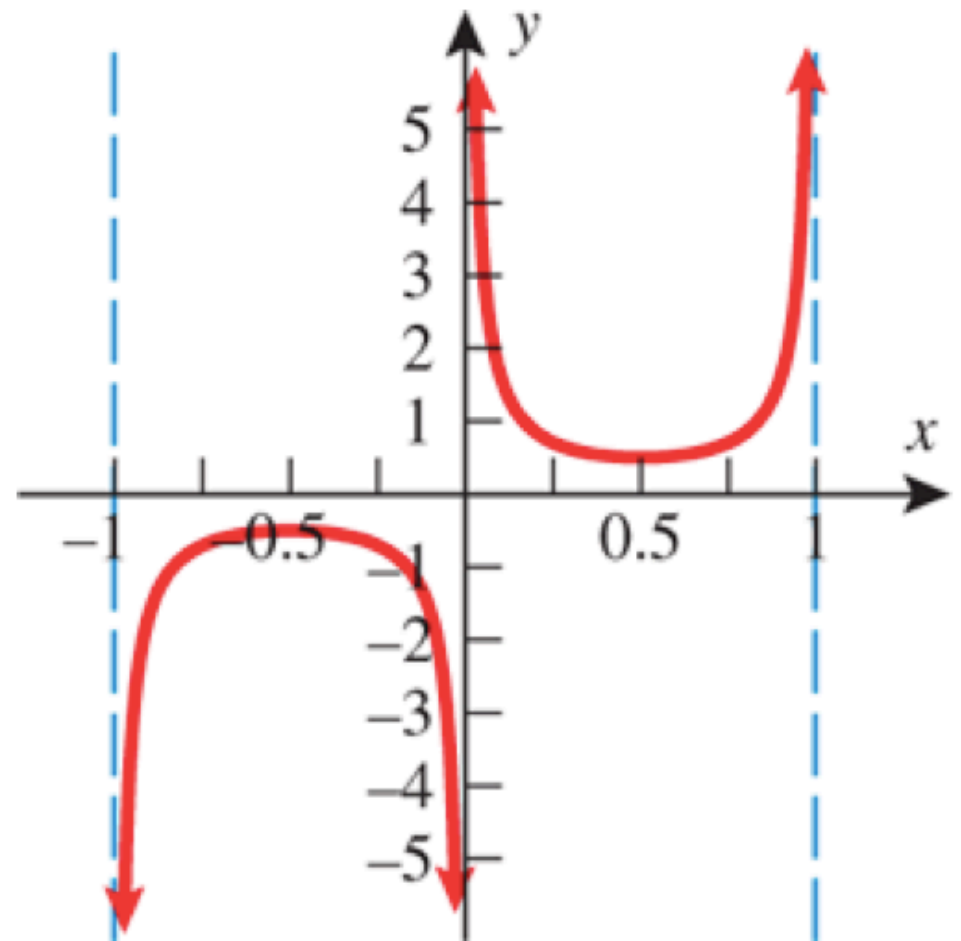
Amplitude  $|A| = \frac{1}{2}$

Period  $p = \frac{2\pi}{\pi} = 2$

Phase Shift

$\pi x = 0$  to  $\pi x = 2\pi$

$x = 0$   $x = 2$



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

**Graphing Function**  $y = A \tan(Bx \pm C) + k$

22/23

Graph  $y = -\tan(x - \frac{\pi}{2}) + 1$  on the interval  $-\pi \leq x \leq \pi$

Period  $p = \frac{\pi}{B} = \pi$

Vertical Asymptote

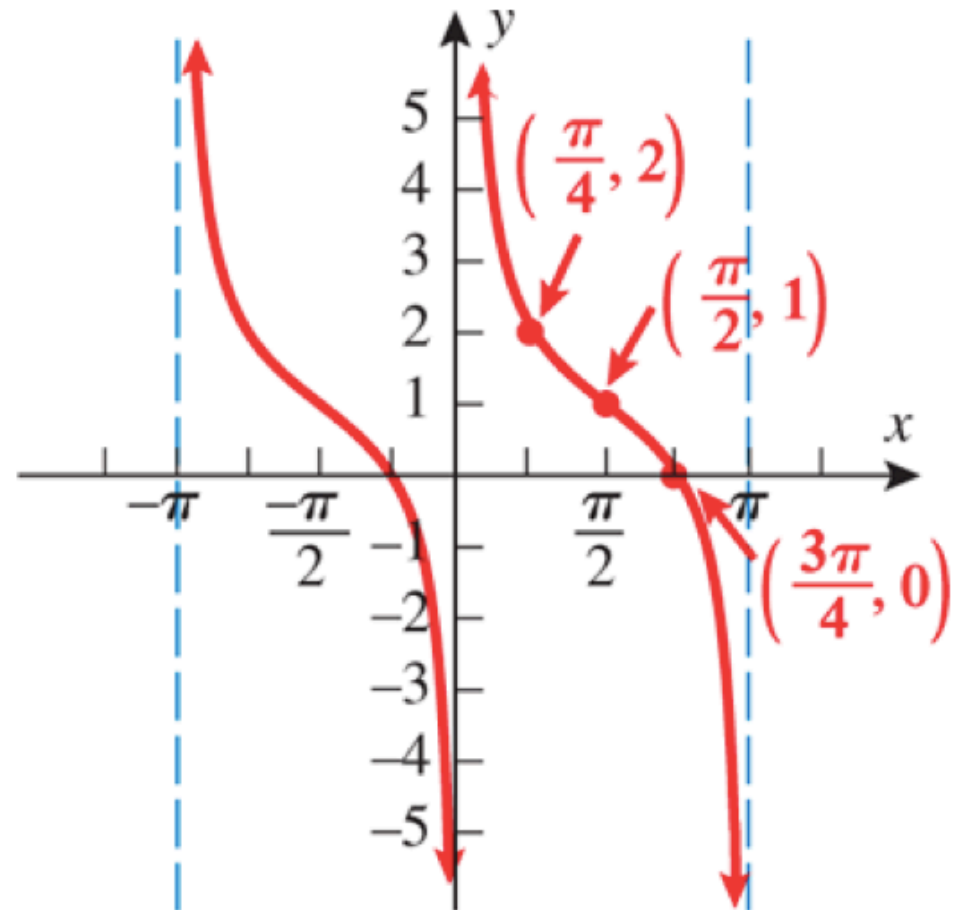
$$x - \frac{\pi}{2} = -\frac{\pi}{2} \quad \text{to} \quad x - \frac{\pi}{2} = \frac{\pi}{2}$$

$x = 0$   $x = \pi$

x-intercept  $x - \frac{\pi}{2} = 0$

$x = \frac{\pi}{2}$

Reflect across x-axis and translate up 1



## 4.3 - Graphing Tangent, Cotangent, Secant, Cosecant

**Graphing Function**  $y = A \csc(Bx \pm C) + k$

23/23

Graph  $y = -\csc(2x - \pi) + 1$  on the interval  $-\pi \leq x \leq \pi$

Use  $y = -\sin(2x - \pi) + 1$  as a guide.

Amplitude  $|A| = 1$

Period  $p = \frac{2\pi}{2} = \pi$

Phase Shift

$$2x - \pi = 0 \quad \text{to} \quad 2x - \pi = 2\pi$$

$$x = \frac{\pi}{2}$$

$$x = \frac{3\pi}{2}$$

Reflect across x-axis and  
translate up 1

